Towards the mass customization of housing: the grammar of Siza's houses at Malagueira

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Abstract. The goal of the described research is an interactive computer system for the design of customized mass housing. Shape grammars are the formalism proposed to systematize the design rules required for such a system. A shape grammar for Alvaro Siza's patio houses at Malagueira, a 1200-unit development still being designed and constructed today, is presented. The grammar is based on the corpus of thirty-five houses designed between 1977 and 1996. The generation of houses in the grammar proceeds by the recursive dissection of rectangles locating four different functional zones (patio, living, services, and sleeping) and the key placement of the staircase. The schematic generations of two existing houses and the detailed generation of a novel one illustrate the grammar.

1 Introduction
The overall goal of the research described in this paper is the development of an interactive computer system for the design of mass housing. Current work is concerned with the development of a shape grammar for the houses designed by the architect Álvaro Siza at Malagueira, near Évora, Portugal. The results of this effort should shed light on the development of more general systems.

Álvaro Siza (1933 – ) is one of the most preeminent and influential contemporary architects. His work has been the focus of numerous studies, but none has tried to understand in depth Siza’s work at Malagueira (1977 – ). Perhaps no other work of Siza is more conceptually meaningful in the context of contemporary architecture than the Malagueira housing development. The neglect of this project is somewhat surprising if one considers that an important part of architectural practice and theory in the 20th century has been concerned with the design of mass housing.

All great masters have addressed in some way or another this design problem in their work. To recall an important few one can mention Walter Gropius’s housing development at Torten (1926 – 28), Le Corbusier’s Domino Houses at Pessac (1926 – 29), and Frank Lloyd Wright’s Usonian houses (1946 – 54). What is common among these projects is the desire to devise a scheme that could be used to generate affordable mass housing, using industrialization as the means for lowering the costs. Gropius used repetition at its extreme. He designed three housetypes for a total of 316 dwellings (Kupferberger, 1974). Le Corbusier, despite the use of repetition, was concerned with variation. He designed four types for a projected 200 dwellings, although only fifty one were built. His concern was essentially placed at the urban scale, as he did not foresee variations within each type. Yet, “if we compare the various interiors evolved by the occupants with Le Corbusier’s original design it is immediately apparent that his conception lent itself to subsequent modification” (Boudon, 1979, page 120). Wright was probably the most concerned with adjusting the design to the households in line with his “concept of houses being as different as their owners” (Sergeant, 1978) and this was clearly expressed in the design of forty-seven different Usonian homes.
Siza's Malagueira is placed at the core of this discussion about housing and it represents a logical development of the previous approaches. This development has its roots in the experiments undertaken in Portugal after the 1974 revolution under the Ambulatory Support to Local residents program, known as SAAL. The program had as one of its desired outcomes the direct participation of future dwellers in the design of their homes (Portas, 1984). In the spirit of the program, designers were expected to work with the future dwellers in order to produce customized dwellings. At Malagueira, as in many other SAAL developments, cooperatives of future dwellers were responsible for promoting the development. The designer was supposed to meet with an assembly of cooperative members to discuss housing types, and then with each individual household to customize its house. Later, assemblies and individual meetings became less frequent because they were time consuming. A similar phenomenon happened at Malagueira. Malagueira was planned as an extension of the city of Évora, and it is a large development encompassing 1200 dwellings. Although the first house was designed in 1977 and built in 1978, design and construction still proceed today. Siza devised a scheme that allowed for the generation of different houses. In fact, over thirty-five different layouts were designed, ranging from one-bedroom to five-bedroom houses. He used this scheme to incorporate into the design process the users' desire for a unique house. The scheme was composed of a set of design rules that were used by Siza or his collaborators to design customized houses. However, despite the potential of Siza's design system, three limitations could be identified. First, it was difficult to convey the rules to other designers because they were never laid down in an explicit way. Second, there were obvious difficulties in representing the universe of solutions by using traditional design media and thus difficulties in conveying them to prospective dwellers. Third, the potential to customize the dwellings was not fully used, despite the ability for generating diverse designs.

This study is an attempt to overcome such limitations and it is based on three arguments: (1) shape grammars can provide the technical apparatus to make Siza's design rules at Malagueira explicit; (2) shape grammars can be used to design customized Malagueira houses; and (3) a computer program encoding such a grammar would allow one to use Siza's design system more effectively. This program, coupled with visualization techniques, can provide a digital framework for customizing the design of mass housing. With such a framework, designers could work with dwellers in the design of their houses and reestablish the dialogue envisaged by the SAAL program. Designing customized housing would be less time consuming and produce better results. In this paper I will illustrate the first two arguments by describing a grammar for the Malagueira houses, by explaining the derivation of existing designs, and by deriving a new design based on the grammar. The third argument is briefly discussed in the concluding section and is the subject of subsequent research.

2 Analytical and original shape grammars

Shape grammar studies can be grouped into two different categories: analytical and original. Analytical grammars have been developed to describe and analyze historical styles or languages of designs by architects no longer living. In fact, after the first grammar was developed to explain a corpus of architectural artifacts, that for Palladian villas (Stiny and Mitchell, 1978), others have been developed with the same purpose over the past twenty years. Analytical studies use a set of existing designs to represent the language—the corpus—and to infer the rules of the grammar. The grammar is then tested by using the rules to generate designs in the corpus, as well as new designs in the language. Original grammars are concerned with the creation of new and
original styles of designs ‘from scratch’. The use of grammars for creative design has not been explored as deeply as the use of grammars for analytical studies.

Although implicit in Stiny and Gips's (1972) paper, such use of grammars was only explicitly addressed by Stiny (1980) when he proposed a program for developing new grammars that was illustrated by Frederick Froebel's kindergarten method of education. Stiny’s program was implemented by Knight who introduced grammars in the design studio. From this experience, Knight highlighted some of the difficulties in using grammars for creative design, which are connected with the difficulty of the “translation of abstract, experimental forms into architectural designs that fit particular design contexts or programmes” (1992, page 48). Solving this difficulty is central to the work described in this paper.

The grammar for Siza's houses at Malagueira follows in the footsteps of the analytical studies mentioned above. Nevertheless, it is a grammar developed for an evolving project by a living architect. To the extent of my knowledge, there has been only one other grammar of this kind: the one on the work of the architect Glen Murcutt (Hanson and Radford, 1986). However, unlike the Murcutt grammar, the Malagueira grammar was developed with Siza’s support and, therefore, it can be seen as a natural extension of Siza’s work at Malagueira. The impact of such a novelty is twofold. First, it is possible to use the architect and the dwellers in addition to existing designs as sources of information to derive the rules of the grammar. Second, it is possible to use the grammar to generate and build new houses in the language. Therefore, the Malagueira grammar is more than a mere analytical grammar aimed at describing a family of designs. But it is not a full grammar developed from scratch to generate entirely new designs. It is reasonable to consider that it spans the gap between analytical and original grammars.

3 Corpus of designs
Three sources of information (drawings, field trips, and interviews) together led to the identification of thirty-five different house designs that constituted the corpus for the grammar. The corpus is not comprehensive, but almost; that is, it does not include all the designed houses, but it includes all the different housetypes. The few houses that were left out have the same functional organization as those that were included and deviate only in small changes of the layout. The included houses were designed between August 1977 and July 1996. Among them are Siza's personal house designed in 1984 and two other customized houses designed by Siza’s collaborator, Nuno Lopes, in 1980 and 1996. The corpus can be classified into two families of houses, depending on whether the yard is at the front or at the back. The frontyard family includes five types. Some of the types include subtypes that differ from one another in details of the layout. Each subtype then has variations in terms of the number of bedrooms, which range from one to five. However, no one-bedroom variation was ever built because of a lack of demand. Figure 1 (see over) shows the plans of the four variations of the first frontyard and backyard houses designed by Siza. Figure 2 (see over) shows a selection of houses in the corpus. Rows show the variation of subtypes, Ab, Ac, Bb, Ca, Cb, Da, and E. The selection aims at giving an idea of the variety of the corpus. The name of a type denotes its functional organization, defined after sequential rule application as described in section 4. Uppercase letters refer to particular patterns in the organization into functional zones (living, sleeping, service, yard, and circulation), whereas lowercase letters correspond to specific spatial arrangements within functional zones. The letter ‘t’ followed by a number identifies the number of bedrooms.
Figure 1. Part of the corpus for the grammar.
Figure 2. Plans, section, and elevations of four variations of a backyard house and a frontyard house.
4 Grammar
As shown by Stiny (1981; 1992), shapes, labels, and weights can be combined to form shape grammars that encode specific languages of designs. Moreover, these grammars can combine several of these components to encode different, but useful, ways of seeing and describing designs, thereby forming a compound grammar. For instance, one can combine grammars to generate plans, elevations, and three-dimensional views into a single compound grammar. The type and number of descriptions or viewpoints depend on the kind of designs captured by the grammar, the purpose one has in developing it, and the desired level of detail. The viewpoints considered in the Malagueira grammar were those used in Siza’s office at the preliminary design stage, when 1:100 scale drawings were manipulated: first-floor, second-floor, and terrace-floor plans; front elevation; and three-dimensional views of the envelope and the enclosed spaces (figure 3).

The Malagueira grammar is a compound, parametric shape grammar defined in the Cartesian product of algebras represented in the following matrix:

\[
\begin{align*}
W & : \langle U_{33}, V_{03} \rangle \\
R & : \langle W_{33}, V_{03} \rangle \\
F_1 & : \langle U_{12}, V_{02} \rangle \\
F_2 & : \langle U_{12}, V_{02} \rangle \\
F_3 & : \langle U_{12}, V_{02} \rangle \\
E & : \langle U_{12}, V_{02} \rangle \\
\end{align*}
\]

The three-dimensional views of the envelope (\(W\)) and the enclosed spaces (\(R\)) are defined in the Cartesian product of algebras \(W_{33}\) and \(V_{03}\). Labelled dots indicating the origin of the referential system and the insertion points of shapes in \(W_{33}\) are the elements in algebra \(V_{03}\). The floor plans and the elevation control the generation of designs, and the three-dimensional views are used only for visualization. The three floor plans (\(F_1 - F_3\)) and the elevation (\(E\)) are defined in the Cartesian product of the algebras \(U_{12}\) and \(V_{02}\). Together, they provide two-dimensional representations of the three-dimensional shapes of Malagueira house designs.

In the product of algebras considered, a rule has the format of a compound rule

\[
\begin{align*}
\begin{bmatrix}
W & : \langle S_{a1}, L_{A1} \rangle \\
R & : \langle S_{a2}, L_{A2} \rangle \\
F_1 & : \langle S_{a3}, L_{A3} \rangle \\
F_2 & : \langle S_{a4}, L_{A4} \rangle \\
F_3 & : \langle S_{a5}, L_{A5} \rangle \\
E & : \langle S_{a6}, L_{A6} \rangle \\
\end{bmatrix} & \rightarrow \\
\begin{bmatrix}
W & : \langle S_{b1}, L_{B1} \rangle \\
R & : \langle S_{b2}, L_{B2} \rangle \\
F_1 & : \langle S_{b3}, L_{B3} \rangle \\
F_2 & : \langle S_{b4}, L_{B4} \rangle \\
F_3 & : \langle S_{b5}, L_{B5} \rangle \\
E & : \langle S_{b6}, L_{B6} \rangle \\
\end{bmatrix}
\end{align*}
\]

meaning that, if certain shapes \(A\) are found in each description, these shapes are replaced by shapes \(B\). Nevertheless, only severely constrained rules require the existence of certain shapes in all the descriptions to be triggered and applied. Most rules require only the presence of certain shapes in some of the descriptions to be triggered and they affect only some of these descriptions when applied. Consider, for instance, a rule to pierce an opening in the front elevation of the first floor. It has the format

\[
\begin{align*}
\begin{bmatrix}
W & : \langle S_{a1}, L_{A1} \rangle \\
R & : \langle S_{a2}, L_{A2} \rangle \\
F_1 & : \langle S_{a3}, L_{A3} \rangle \\
F_2 & : \langle S_{a4}, L_{A4} \rangle \\
F_3 & : \langle S_{a5}, L_{A5} \rangle \\
E & : \langle S_{a6}, L_{A6} \rangle \\
\end{bmatrix} & \rightarrow \\
\begin{bmatrix}
W & : \langle S_{b1}, L_{B1} \rangle \\
R & : \langle S_{b2}, L_{B2} \rangle \\
F_1 & : \langle S_{b3}, L_{B3} \rangle \\
F_2 & : \langle S_{b4}, L_{B4} \rangle \\
F_3 & : \langle S_{b5}, L_{B5} \rangle \\
E & : \langle S_{b6}, L_{B6} \rangle \\
\end{bmatrix}
\end{align*}
\]

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Figure 3. Viewpoints and algebras considered in the compound grammar.
where $S, L_0$ are empty labelled shapes. It is triggered when certain shapes are found on the three-dimensional envelope, first-floor plan, and the elevation; and it affects only these views when applied.

In the Malagueira grammar the plan of the first floor drives the generation of designs in the grammar. The layout of upper floors is, to a considerable extent, constrained by decisions made on the first floor, for structural constraints. The elevation is also determined by the layout of the floors. This dependency is encoded into the grammar through the use of sequential, parallel grammars, one for each floor and another for the front elevation, as shown in figure 4.

The derivation of a design in the grammar goes through three successive stages: defining the first floor ($F_1$), defining the second floor ($F_2$), and defining the terrace ($F_3$). As the generation of the first floor proceeds, labels are placed on the second floor and on the elevation ($E$). When the generation of the first floor finishes, a state label changes, thereby activating the generation of the second floor, which proceeds using the previously placed labels as beacons. The articulation between the generation of the second floor and the terrace works in a similar fashion. Each of these stages, in turn, includes several steps. For instance, the stages of the first floor are locating functional zones, locating the staircase, dividing functional zones (into rooms), introducing details (fireplaces, chimneys, etc), and introducing openings. The separation into steps is merely analytical, as there are no state labels to go from one step to another, like those used to change stage.

Key: $F_1$—first floor; $F_2$—second floor; $F_3$—terrace; $E$—elevation; $S$—start; $Z$—locate functional zones; $C$—locate circulation scheme; $R$—divide zones into rooms; $D$—introduce details; $O$—introduce openings; $T$—terminate.

**Figure 4.** Use of sequential, parallel grammars in the derivation of a Malagueira house. Dark shaded areas identify the currently active grammars (viewpoint), light shaded areas identify passive grammars, and nonshaded areas identify nonactive grammars. Letter symbols identify steps of the derivation, and arrow symbols identify the input of labels by the active grammar into the passive grammar.

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**Figure 5.** Urban context of Malagueira house plots: a—street only at the front—typical lot (longitudinal symmetry); b—street at the front and on the side (no symmetry); c—street at the front and at the back (longitudinal and transverse symmetry); d—street at the front, at the back, and on the side (transverse symmetry). The numbers on the lower right corner of plots refer to the counting on figure 10.
4.1 Context
In the Malagueira grammar, the initial shape is a rectangle with a label ‘Lot’ representing the 8 m × 12 m plot. The plots are clustered together to form housing blocks. In most cases these blocks are rectangular, but they might take other forms to adjust to the shape of curvilinear roads. As a result, in typical plots all but the front edge border other plots, but in some plots edges other than the front edge might border a street, within certain limitations. For instance, detached plots are not permitted. The type of surroundings, illustrated in figure 5, defines the urban context of the lot, and impacts the functional organization of its house by restricting the number of elevations with openings.

4.2 Composition
In brief, the compositional principles behind the generation of a Malagueira design are based on the manipulation of rectangles representing rooms by means of rules for dissecting, connecting, and extending rectangles, as well as rules for assigning and changing the functions associated with them. To make it easier for the reader to understand the compositional principles of the grammar, a very simplified set of shape rules is shown in figure 6 (see over). In this simplification, rules include only the two-dimensional plan component and lines represent walls. The labels fn(n = 1, 2) denote the functions of the rooms that the rectangles represent. The dot • is a label that identifies the last line placed and indicates on which side the next dissection may occur: on both sides (rule A) or only one side (rule B). Perpendicular dissection is the primary compositional principle. In rules A and B, dissections are perpendicular to the bigger edge of the rectangle, whereas in rule C it is perpendicular to the smaller one. Rule D deletes the label •, preventing further dissections. Rules E and F concatenate two adjacent rectangles to form a larger room. Rule G extends a room at the expense of an adjacent one. Rule H assigns a function to a room. Rule I permutes the function of two adjacent rooms. In addition to rectangular dissections, the Malagueira grammar includes some diagonal dissections. Rules J, K, and L dissect a rectangle by tracing lines that establish 30° or 60° angles with its edges, thereby generating right triangles, trapezoids, and pentagons. To deal with these shapes other rules are required in the grammar. Rule M dissects a right triangle; rules N, O, P, and Q dissect trapezoids; and rules R, S, and T dissect pentagons. There are no rules to dissect triangles, trapezoids, and pentagons with diagonal lines in order to prevent further deviation from rectangular shapes. Rules U, V, and W concatenate rectangles with adjacent triangles, trapezoids and pentagons. Siza used diagonal dissections only in the first two designs because dwellers did not like the nonrectangular rooms they generated. The generation of basic layouts with rules A–N comprises two steps. In the first step, the lot is divided into the four functional zones—patio, living, service, and sleeping—thereby obtaining a basic pattern, and then a staircase is added thereby defining a stair pattern and the housetype. In the second step, these zones are divided into rooms to obtain the layout. The steps involved in the definition of the functional organization of the first floor are shown in figure 7 (see over). The diagram takes the form of a tree in which nodes represent the state of the design and arcs represent the application of rules. The tree illustrates how the application of rules to allocate functional zones generates the five basic patterns behind the houses in the corpus. It also shows how the different types in the corpus derive from these patterns by a different application of the rule to locate the staircase. Finally, it shows that subtypes differ from one another in small variations of the layout caused by different applications of the rules for dividing zones. Figure 8 (see over) shows the first-floor derivations of an existing frontyard house (subtype Ab) and a backyard house (subtype Bb) with the rules described above.
Figure 6. Simplified grammar rules.
4.3 Function

As a way of facilitating the spatial and the mathematical understanding of the grammar rules, a detailed rule is shown in figure 9 (see over). The parametric rule shown is ‘rule 9: dissecting the outside zone into yard and sleeping zones’. As with all the rules in the grammar, it has a shape part (S), a label part (L), and a set of control conditions on functional and dimensional aspects. As shown above, the shape part is used to specify the compositional principles. In rule 9 the shape part is generic as it is shared by several dissecting rules. The label part is used mainly to deal with the contextual requirements and the functional strategy involved in the generation of Malagueira houses. The generic format of the label part of a dissecting rule has the form \( R_i : \langle F_n; f_b, f_r, f_f, f_l; f; Z \rangle \rightarrow \langle F_n; f_b, f_r, f_f, f_l; f_l; f_2; Z - \{f_1, f_2\} \rangle \). In this expression \( R_i \) is the rule number, such as \( R_9 \) for rule 9. The label \( F_n \), \( n \in \{1, 2, 3\} \), indicates the
stage of the derivation to which the rule applies. In rule 9, \( n = 1 \) — 1st floor, which means that the rule applies only to the generation of the first floor. The labels \( fb \), \( fr \), \( ff \), and \( fl \) identify the functions associated with adjacent rectangles at the back, right, front, and left side of the rectangle currently considered for dissection. These labels, coupled with conditional statements are used to express adjacency requirements, thus

Subtype Ab

Subtype Bb

Figure 8. Simplified derivations of the first-floor functional organization of subtypes Ab and Bb. The symbol * following a rule letter (A, B, etc) means that the rule was applied several times.
determining the topology. In rule 9, fl = li—living room, which restricts rule application to finding in the evolving design a rectangle adjacent to the living room. The labelled point • identifies the function currently associated with the rectangle being dissected (in rule 9, f = o — outside zone), whereas labels f1, and f2 identify the function of the resulting rectangles (in rule 9, f1 = ya—yard zone, f2 = sl—sleeping zone). Together with conditional statements these labels specify the type of possible functional dissections (in rule 9, ya, sl ∈ Z—the set of required zones). Each time a dissecting rule is to be applied, the zones allocated by the rule are retrieved from the set of required zones. Once the rule has been applied and the zones have been created, they are deleted from this set to prevent further allocations of such zones in subsequent steps of the generation (in rule 9, Z = {ya, sl}). Other control conditions specify dimensional
constraints (for example, \( w_m < w_{wl} < w_x, l_m < l_i, l_m < l_2 \)). Namely, they assure that the dimensions of the zone to be dissected are such that they permit the allocation of the intended zones. In other words, allocation takes place only when the dimensions of the new zones can be within a certain range. This range was established after dimensional analysis of the existing designs. The result of allocating functional zones is a basic pattern of the first-floor layout. The allocation of rooms within zones, including the allocation of the staircase, proceeds in a similar fashion. The rooms in the set of required rooms are specified before the generation starts and form the housing program. Computation terminates when all the rooms have been allocated and the functional organization has been defined. The range of dimensions for rooms follows existing regulations, as the analysis of drawings showed that Siza respected the regulations, without further constraints.

Because the structural system used is a load-bearing wall system, the generation of the first floor largely determines the generation of the second floor. As the generation of the first floor proceeds, labels are placed on the second floor and on the elevation. When the generation of the first floor is finished, the generation of the second floor begins, using these labels as beacons. Consider rule 9 again. Label \( Q_2 \) is placed on the second floor to indicate where the dissection took place. In this label, the exponent ' indicates that it is a second-level dissection. (The first-level dissection is that of the lot into outside and inside zones.) The index 2 refers to the floor. Labels \( f^1 \) and \( f^2 \) determine the kind of the zones that can be created on the second floor, in terms of indoor or outdoor, depending on the kind of dissected zone. In this label, the exponent ' indicates it refers to the second floor. If the dissected zone is an outside zone, two things can happen. If the resulting zone \( f_n, n \in \{1, 2\} \), is a yard zone, then the label \( f^n, n \in \{1, 2\} \), becomes \( o \), meaning that the zone on the second floor will be an outside zone. If the resulting zone is a zone other than the yard zone, then it becomes \( i \). Finally, if the dissected zone is an inside zone, then the label becomes \( i \). Other circumstances will determine to which kind of rooms \( i \) and \( i' \) zones will give origin (terraces or indoor rooms). In the case of rule 9, \( f^1 = o \) and \( f^2 = i' \). See figure 12 (below) for an illustration of these labels operating.

4.4 Structure
The structural system of Malagueira houses is based on load-bearing walls made of concrete blocks. This system impacts the design by constraining the thickness and height of walls and the span between load-bearing walls, as well as the need for aligning load-bearing walls on upper floors with those on lower floors. Such structural constraints are encoded into the grammar. Constraints on thickness and span are encoded into dissecting rules by placing conditions on the thickness and location of dissecting walls. Consider, for instance, rule 9 which divides the inside or outside zones into functional zones. Because the dissecting wall is a load-bearing wall, it requires its thickness to be 0.2 m, and it limits its location so that a maximum span of 6 m is respected. Constraints on wall alignment are encoded into dissecting rules by requiring dissections on upper floors to be aligned with dissections on lower floors through the placement of appropriate labels as described in the previous section. The influence of the span between load-bearing walls on the thickness of the slab they support is encoded by using the span value to retrieve the required thickness from a table (not shown) when placing the slab, later in the derivation. Constraints on wall height are encoded by choosing a standard value that is a multiple of the height of the concrete block and higher than the minimum floor height. Other structural constraints operate on rules for concatenating adjacent rooms and on rules for piercing openings to prevent the deletion of large extensions of load-bearing walls.
4.5 Universe of solutions

The design of the grammar faced a paradox. On the one hand, one needed a grammar that generated a large set of design solutions to increase the potential for generating customized designs. On the other hand, one wanted to make sure that the grammar generated only designs in the Malagueira style and that a solution could be found in practical time. To overcome this paradox, three steps were followed in the design of the grammar. The first step was to develop the exhaustive set of rules that could be derived from the compositional principles of dissecting and concatenating rectangles. The second step was to limit such an exhaustive set of rules whenever it seemed that it would oppose Siza’s compositional rules. The third step was to generate new designs with a more open set of rules and then ask Siza whether he considered them to be in the grammar. The universe of design solutions is partially illustrated in figure 10. Patterns with a dot • underneath correspond to houses designed by Siza.

Figure 10. The potential universe of solutions in the Malagueira grammar. Each level in the tree represents a single step in the derivation. Because of space limitations, only the steps required for the derivation of dimensioned stair patterns are shown, and only one branch is expanded to the level below. The symbol • identifies patterns that correspond to houses designed by Siza. The symbol ○ identifies patterns considered in the final version of the grammar but not used by Siza.
Patterns with a dot \( \circ \) are other patterns considered in the grammar following a more open interpretation of Siza’s rules.

To count the exact number of solutions is nearly impossible, but it is feasible to form an idea by estimating the upper and lower bounds of the interval that corresponds to steps 1 and 3 mentioned above, particularly by counting the number of different patterns that can be generated in the initial steps of the derivation. Considering that the Malagueira urban blocks are defined within a grid of rectangular plots, each plot is bordered by 8 other plots. The front three plots form a street, but the other five neighboring plots can either be occupied by a house or become part of a street. Therefore, there are 32 \( (2^5) \) potential context patterns \( (CP) \). After the symmetrical patterns have been eliminated, this figure is reduced to 20. The application of urban planning rules further reduces this number to 8. By considering that the corner lots have no impact on the functional organization of the central lot, the number of context patterns is reduced to 4. The rules for allocating functional zones divide the lot into 4 zones, thereby defining 8 geometric patterns. For each of these patterns, 24 topological \( (TP) \) can be obtained by assigning functions to each zone \( (4 \times 3 \times 2 \times 1) \). The term topology is used to refer to the articulation of functional spaces. Symmetrical patterns cannot be eliminated when placed in the same nonsymmetrical urban context because they yield different housing solutions. This means that there are 192 \( (8 \times 24) \) topological patterns if the urban context is not symmetrical, 144 if it has longitudinal symmetry, 96 if it has transversal symmetry, and 52 if it has both types of symmetry. However, only 128 \( (8 \times 16) \) patterns in the first case were considered in the grammar. Consider now patterns with the staircase included. Figure 10 shows all stair patterns \( (ST) \) obtained by locating the two types of staircase (U-shaped, or L-shaped or I-shaped) in the living zone in all possible ways. There are 24 such patterns, meaning that there are 96 \( (4 \times 24) \) potential stair patterns for each topological pattern. If we follow a stricter interpretation of Siza’s rules, this number is considerably reduced. None of the staircases can be located in the yard zone; an L-shaped or I-shaped staircase is located in the living zone in such a way as to minimize circulation; and a U-shaped staircase is transversally located in the sleeping or service zones. Therefore, there are 4 possible ways of locating L-shaped or I-shaped staircases in the living zone, and 8 possible ways of locating U-shaped ones in the service and sleeping zones. The total number of stair patterns in these circumstances is 20 \( (4 + 8 + 8) \) for each topological pattern. The upper and lower bounds of the universe of solutions, obtained by multiplying \( CP \times GP \times TP \times SP \), is 360,960 \( (20 \times 8 \times 24 \times 94) \) and 10,240 \( (4 \times 8 \times 16 \times 20) \), respectively.

The dimensioning of zones has not so far been considered. For each stair pattern there are, in fact, many distinct dimensioned patterns. There are two possible ways of positioning the line that dissects the lot into inside and outside zones \( (6 \text{ m and } 7 \text{ m from the front border}) \). The positioning of the walls that subdivide the inside and outside zones is more complicated to take into account the fact that Siza did not restrict it in a similar way. For the sake of the discussion, assume that they can be placed at 0.05 m intervals, and recall that the minimum width of rooms \( (2.2 \text{ m}) \) is respected. The number of dimensioned patterns varies considerably, depending on the geometric pattern, but is very large. For instance, there are 2816 \( (2 \times 44 \times 32) \) dimensioned patterns just for stair pattern 3. If the division of zones into rooms is considered too, the number of different solutions becomes even larger. Then, we would have to consider the result of using rules for diagonal dissections, rules for concatenating spaces, let alone the rules for detailing spaces, and the rules for making openings. The universe of potential solutions is, thus, in the order of millions.
4.6 Rules
To make it possible for the reader to grasp the complexity of the grammar, the set of rules is shown in figure 11 (over). Because of space limitations, only some of the rules used in the generation of the first floor are shown. Rules for diagonal dissections, rules for introducing openings into diagonal walls, and some of the rules for introducing chimneys are also excluded. Also, representation is still further simplified, as three-dimensional views and empty two-dimensional views are not included. The rules are explained below, following the division into stages and steps referred to above.

Stage 0: introduce initial shape
A single rule, rule 0, applies at this stage. This rule introduces the initial shape and adds a set of labels. The initial shape is a 8 m × 12 m rectangle representing the lot. The labels s and h around the edges of the lot tell whether they border a street or a house and express the urban context. The labels Q1 specify the two ways in which a lot can be split into two halves in a subsequent step of the computation to allocate the patio and the house. F0 is a state label placed at the origin to indicate the current stage of the derivation.

Stage 1: define the first floor
As mentioned in a previous section, the definition of the first-floor plan goes through six different steps: start (S), locate functional zones (Z), define circulation scheme (C), divide zones into rooms (R), introduce details (D), introduce openings (O), and terminate.

Step 1.1: start. Rules 1–4 apply at this stage. Rule 1 introduces the slab, a 0.2 m thick box that corresponds to the standard difference in level between the ground floor and the street. Rule 2 creates the walls that enclose the floor. Rules 3 and 4 increase their thickness to 0.2 m when the wall borders the street. The condition $p(s) \in \{ff, fb\}$ means that the rule applies if the position of the label s is at the front or at the back of the lot.

Step 1.2: locate functional zones. Rules 5 and 6 decide whether the outside zone (identified by label o) is going to be located at the front or at the back of the lot. Label Q1, inherited from rule 1, determines the location of the dissecting, load-bearing wall. There are two possible locations: 6 m (rule 5) and 7 m (rule 6) away from the front border of the lot. Label Q2 marks where the dissection was made so that this information can be used in the derivation of the second floor. Rule 7 then applies to link the backyard to the street by creating a 1.1 m wide corridor (co). Rules 8 and 9 apply to locate the living zone (li) by dissecting the inside zone into living zone and sleeping (sl) or into living zone and service zone (se). Rules 10 and 11 locate the remaining zone by dissecting the outside zone, thereby determining the definite location of the yard (y). Rule 12 determines that the outside zone becomes the yard by changing the label o into y, thereby preventing further dissections. This can be applied only when the number of desired bedrooms is two ($tn = t2$). The location of the dissecting wall is determined by structural and functional constraints. Label Q’2 marks where the zone was actually dissected so that this information can be used in the derivation of the second floor. The symbol Z represents the set of required zones. Each time a rule is applied, the allocated zones are subtracted from this list.

Step 1.3: define circulation scheme. Two sets of rules apply at this stage. The first set includes rules 13 and 14, which locate the main entrance to the house from the yard or the external corridor, near the wall that separates the living from the adjacent zone. They differ on the placement of label e, which establishes the circulation axis; in rule 13 this axis is parallel to the yard, whereas in rule 14 it is perpendicular.
Figure 11. Partial set of detailed rules. Because of space limitations, only the rules used in the generation of the first floor are shown. Rules for diagonal dissections and rules for introducing chimneys on the first floor are also not shown. A more complete set of rules may be found at http://www.civil.ist.utl.pt/~jduarte/epb/figure11.pdf.
R5: Locate patio at (0, 6) m

R6: Locate patio at (0, 7) m

R7: Locate functional zones

R8: Outside zone becomes patio zone

Figure 11 (continued)
R13: Locate main entrance

R14: Locate main entrance

On function: \( f = li \lor fb = y \lor fb = co \)

R15: Locate L-shaped staircase in the living zone

R16: Locate I-shaped staircase in the living zone

On function: \( f = li \lor fb = y \lor fb = co \)

R17: Locate I-shaped staircase in the living zone

R18: Locate U-shaped staircase in the sl or se zones

On function: \( F1, fb, fr, ff, fe, li; li; st; f \neq o \)

On function: \( f = s1 \lor f = se \land fb = li \)

Figure 11 (continued)
R19 – 20: Dissect perpendicular to smaller edges

On dimension: wmin < w1; t = 0.1 m; p = 1 m
On function: (patio) R19: \((F1, fb, fr, ff, fl, se; se, ki)\)
(living) R20: \((F1, fb, fr, ff, li, li, ci)\)

R48 – 57: Terminate division of zones

On function:
R48: \((F1, fb, fr, ff, li, y, y)\); \(R^\ast se = \emptyset \land R^\ast si = \emptyset\)
R49: \((F1, fb, fr, ff, li, li, fr)\);
R50: \((F1, fb, fr, ff, li, sc; ki)\); \(R^\ast se = \emptyset\)
R51: \((F1, fb, fr, ff, li, se; ts)\); \(R^\ast = \emptyset\)
R52: \((F1, fb, fr, ff, li, se; pa)\); \(R^\ast se = \emptyset\);
\(fb \in \{ki, li\} \Rightarrow \neg f \in \{ki, li\}\); \(tr \in \{ki, li\} \Rightarrow \neg f \in \{ki, li\}\)
R53: \((F1, fb, fr, ff, li, se; la)\); \(R^\ast se = \emptyset\);
\(fb \in \{ki, li\} \Rightarrow \neg f \in \{ki, li\}\); \(tr \in \{ki, li\} \Rightarrow \neg f \in \{ki, li\}\)
R54: \((F1, fb, fr, ff, li, se; y)\); \(R^\ast se = \emptyset,\)
\(\exists n: fn = y, n \in \{b, t, f\}\)
R55: \((F1, fb, fr, ff, li, se; ba)\); \(R^\ast si = \emptyset\)
R56: \((F1, fb, fr, ff, li, sl, ci)\); \(R^\ast si = \emptyset\)
R57: \((F1, fb, fr, ff, li, sl, y)\); \(R^\ast si = \emptyset\)
\(\exists n: fn = y, n \in \{b, t, f\}\)

Figure 11 (continued)
R59: Categorize adjacent rooms

\[ \begin{align*}
\text{R59: } & \text{Categorize adjacent rooms} \\
\end{align*} \]

On dimension: \( p \geq 0; p' \geq 1.2 \text{ m}; p'' \geq 0 \)

On function:
\[ (F_1, \text{tb}, \text{fr}, \text{tl}, \text{st}; \text{st}; \text{cl}) ; \\
F = F_1 \Rightarrow [t_1 = t_2 \lor (t_1 \neq t_2 \Rightarrow t_1, t_2 \in R_i \lor \exists n : n = n_i) , \\
i \in \{y, ll, se, sl\}, n \in \{1, 2\}] \\
F = F_2 \Rightarrow [t_1 = t_2 \lor (t_1 \neq t_2 \Rightarrow 3n : n = n_i)] \]

R60: Locate laundry

\[ \begin{align*}
\text{R60: Locate laundry} \\
\end{align*} \]

On function:
\[ (F_1, \text{tb}, \text{fr}, \text{tl}, \text{st}; \text{st}; \text{cl}) ; \text{tn} = t_2 \]

R61: Permute functions

On function:
\[ (F_1, \text{tb}, \text{fr}, \text{tl}, \text{st}; \text{st}; \text{cl}) ; \text{tn} = t_2 \]

R62: Introduce chimney (rules 63–67 not shown)

\[ \begin{align*}
\text{R62: Introduce chimney (rules 63–67 not shown)} \\
\end{align*} \]

On dimension: \( t = 0.1 \text{ m}; 1.0 \leq p \leq 1.2 \text{ m} \)

On function:
\[ p(h) \in \{t, \text{fr}\} \]

R74–76 Detail stairs

\[ \begin{align*}
\text{R74–76 Detail stairs} \\
\end{align*} \]

On dimension: \( t = 0.25 \text{ m} \)

R77: Adjust the patio wall height

\[ \begin{align*}
\text{R77: Adjust the patio wall height} \\
\end{align*} \]

On dimension: \( h = 1.5 \text{ m} \)

R78: Adjust the patio wall height

\[ \begin{align*}
\text{R78: Adjust the patio wall height} \\
\end{align*} \]

On dimension: \( h = 1.5 \text{ m} \)

Figure 11 (continued)
R79: Pierce front façade openings

R80: Pierce front façade openings

On dimension: \( w \in \{1.05, 1.10\}; h_1 \in \{0.0, 0.2\}; h_2 \in \{2.00, 2.07\}\)

R81: Erase axis of symmetry

R82: Pierce exterior opening on the axis of symmetry

On dimension: \( w \in \{1.05, 1.10\}; h_1 \in \{0.20, 0.35, 0.75\}; h_2 \in \{2.00, 2.07\}; h_3 \in \{0.0, 0.2\}; h_4 \in \{2.00, 2.07\}\)

R83: Erase invisible patio opening

R84: Erase invisible patio opening

On dimension: \( h = 1.5 \text{ m} \)
On function: \( f_f \in \{ya, te\}; F_n \in \{1, 2\}\)

R85: Erase invisible patio opening

R86: Pierce an exterior opening in the middle of a room wall on the front façade

On dimension: \( w \in \{1.05, 1.10\}; h_1 \in \{0.0, 0.2\}; h_2 \in \{2.00, 2.07\}\)

On function: \( f_f \in \{ya, te, s\}; F_n \in \{F_1, F_2\}\)

Figure 11 (continued)
R87: Pierce an exterior opening in the middle of a room wall

\[
\begin{align*}
\text{On dimension: } w & \in \{1.05, 1.10\} \\
\text{On function: } \{\text{ya, te, s}\}, \forall fb
\end{align*}
\]

R88: Pierce entrance door in the middle of the patio

\[
\begin{align*}
\text{On dimension: } w & \in \{1.05, 1.10\} \\
\text{On function: } \forall ff, fr
\end{align*}
\]

R89: Pierce the entrance door

\[
\begin{align*}
\text{On dimension: } w & \in \{1.05, 1.10\} \\
\text{On function: } \{\text{li, st, ci}\}, \forall fb
\end{align*}
\]

R90: Pierce an exterior opening abutting a room wall

\[
\begin{align*}
\text{On dimension: } w & \in \{0.7, 0.8, 0.9\} \\
\text{On function: } \forall ff, tr
\end{align*}
\]

R91: Pierce exterior openings in a row on the patio wall

\[
\begin{align*}
\text{On dimension: } w & \in \{1.05, 1.10\}; t = w/2
\end{align*}
\]

R92: Pierce an exterior opening facing another

\[
\begin{align*}
\text{On dimension: } w & \in \{0.7, 0.8, 0.9\}
\end{align*}
\]

R93: Pierce an exterior opening facing another

\[
\begin{align*}
\text{On dimension: } w & \in \{1.05, 1.10\}
\end{align*}
\]

R94: Pierce an interior door next to a wall

\[
\begin{align*}
\text{On dimension: } w & \in \{0.7, 0.8, 0.9\}
\end{align*}
\]

R95: Pierce an interior opening facing an exterior opening

\[
\begin{align*}
\text{On function: } \forall fb
\end{align*}
\]

R96: Pierce an interior opening in the middle of a wall

\[
\begin{align*}
\text{On dimension: } w & \in \{0.7, 0.8, 0.9\}
\end{align*}
\]

R97: Pierce an interior opening between kitchen and transitional space

\[
\begin{align*}
\text{On dimension: } w1 = 0.9; w2 = 0.3
\end{align*}
\]

R108: Erase labels

Figure 11 (continued)
The other set of rules includes rules 15–18. Each of these rules places the staircase in a way that it overlaps label e to minimize circulation. Rule 15 places an L-shaped staircase in the living zone, whereas rules 16 and 17 place an I-shaped staircase. Rule 18 places a U-shaped staircase in the service or in the sleeping zones.

The stairs always have fourteen steps; the treads are 0.25 m deep and the risers depend on the floor height. Twelve of the steps constitute the body of the stairs, which is bounded by runaways forming the remaining steps. If the living zone is not large enough, the linear staircase takes the form of an L-shape (rule 15), or invades into the neighbouring zone (rule 16). The minimum length of the long tail of the L-shaped staircase is restricted to ten steps in order to guarantee that a person does not hit the ceiling when climbing the stairs. All the rules for placing the staircase adjust the dimension of the zone in which the staircase is placed in order to comply with the rule for stair design just described.

**Step 1.4: divide zones into rooms.** Computation at this stage is based either on the recursive dissection of zones to create rooms or on the connection of rooms that are functionally related to form larger rooms across the previously defined zones. The set of rooms that can be included in the program of a Malagueira house is shown in table 1. The breakdown of rooms into obligatory and optional sets is shown in the same table.

**Table 1. Rooms by zone.**

<table>
<thead>
<tr>
<th>Floor</th>
<th>Zone</th>
<th>Rooms</th>
<th>Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both</td>
<td></td>
<td>possible: ( R = R' \cup R^s )</td>
<td>( R = R' \cup R' \cup R^{se} \cup R^{sl} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>required ( R' = R'y \cup R'li \cup R'^{se} \cup R'^{sl} )</td>
<td>( R' = R'li \cup R'^{se} \cup R'^{sl} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>optional ( R'' = R''li \cup R''^{se} \cup R''^{sl} )</td>
<td>( R'' = R''li \cup R''^{se} \cup R''^{sl} )</td>
</tr>
<tr>
<td>First</td>
<td>pa</td>
<td>patio</td>
<td>Ry = R'y = {y} \cup R'^{y} = \emptyset</td>
</tr>
<tr>
<td>li</td>
<td>living</td>
<td>lr</td>
<td>living closet ( Rli = R'li = {li} \cup R'^{li} = {cl} )</td>
</tr>
<tr>
<td>se</td>
<td>service</td>
<td>ki</td>
<td>kitchen ( Rse = R'^{se} = {ki} \cup R'^{se} = {ts, la, pa} )</td>
</tr>
<tr>
<td>la</td>
<td>pantry</td>
<td>ts</td>
<td>transitional space</td>
</tr>
<tr>
<td>sl</td>
<td>sleeping</td>
<td>be</td>
<td>bedroom ( tn = t2 \Rightarrow Rsl1 = R'^{sl1} = {be1, be2, ba} \cup R'^{sl2} = {cl} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ba</td>
<td>bathroom ( tn \neq t2 \Rightarrow R = R'^{sl1} = {be1, ba} \cup R'^{sl2} = {cl} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ci</td>
<td>circulation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>st</td>
<td>stairs*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>co</td>
<td>corridor^b</td>
</tr>
<tr>
<td>Second</td>
<td>be</td>
<td>bedroom</td>
<td>( tn = t1 \Rightarrow Rsl1 = \emptyset )</td>
</tr>
<tr>
<td>ba</td>
<td>bathroom</td>
<td>( tn = t2 \Rightarrow Rsl1 = \emptyset \cup R = R'^{sl1} = {be1, be2, ba} \cup R'^{sl2} = {cl} )</td>
<td></td>
</tr>
<tr>
<td>cl</td>
<td>closet</td>
<td>( tn = t3 \Rightarrow Rsl1 = R'^{sl1} = {be2, be3, ba} \cup R'^{sl2} = {cl} )</td>
<td></td>
</tr>
<tr>
<td>ci</td>
<td>circulation</td>
<td>( tn = t4 \Rightarrow Rsl1 = R'^{sl1} = {be2, be3, ba} \cup R'^{sl2} = {cl} )</td>
<td></td>
</tr>
<tr>
<td>st</td>
<td>stairs</td>
<td>( tn = t5 \Rightarrow Rsl1 = R'^{sl1} = {be2, be3, be4, ba} \cup R'^{sl2} = {cl} )</td>
<td></td>
</tr>
<tr>
<td>te</td>
<td>terrace</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

^a The stairs can be included in any of the interior zones.

^b The corridor to the backyard does not belong to any of the zones.
The final set of rooms in a particular case depends on the desired program. These sets are then used to control the derivation in order to guarantee that a house that fits a given program will be generated. \( R' \) is the set of obligatory rooms and \( R'' \) is the set of optional rooms. The computation starts with \( R' \), the set of obligatory rooms, including all the desired rooms. Each time a dissecting rule is applied to create a new room it removes this room from the set of obligatory rooms. The computation stops when this set becomes empty or it continues until the set of optional rooms becomes empty too. The exact location of a dissection is determined by the width, length, and area values permitted for the room being allocated. The upper and lower bounds of the range of these values are prescribed by regulations.

There are five groups of rules that can be applied at this stage: dividing, extending, assigning, connecting, and permuting rules. Dividing rules (rules 19 – 32) divide a functional zone into rooms. Of these rules, only the rules to divide the living and the patio zones to create a circulation area consist of dissections perpendicular to the smaller edges of the rectangular rooms (rules 19 – 20). All the remaining rules make dissections perpendicular to the larger edges. Extending rules (rules 33 – 47) divide a zone to extend an adjacent zone. Rules 33 – 38 make perpendicular dissections, whereas rules 39 – 47 (not shown) make 30° and 60° dissections. Assigning rules (48 – 57) are used to create the last required room, or an optional room, out of the space that remains after recursive application of dissecting rules. The rule that cuts the yard out of the yard zone has the additional feature of lowering the ground level. Application of these rules prevents further application of the dissecting rules. Rule 58 is a dissecting rule that turns the space beneath the staircase into a closet. Rule 59 connects any two rooms of any shape that share, at least, a 1.2 m wall, provided that they have the same or related functions. Rule 60 adds an external cubic laundry to a concave corner of the yard that is adjacent to the service zone. Rule 61 is a permuting rule that applies if the house has two bedrooms. This rule turns the kitchen into a bedroom and the transition space into the kitchen, thereby avoiding the need for a second floor.

**Step 1.5: introduce details.** Four sets of rules apply at this stage. The first set (rules 62 – 67) creates chimneys. All these rules can be applied to the kitchen; in addition, rules 64 – 67 (not shown) can be used for the living room as well. The second set of rules (rules 68 – 73) adjusts the thickness of the walls depending on their location. Rules 68 – 72 increase the thickness of exterior walls towards the outside from 0.075 m to 0.2 m. Rule 68 increases the wall between the living room and the patio to accommodate window shutters when these are opened. Rule 73 decreases the thickness of interior walls from 0.2 m to 0.075 m, if the span is smaller than 2 m. The third set of rules (rules 74 – 76) includes rules to complete the design of the stairs. The fourth set (rules 77 – 78) decreases the height of the patio walls to 1.5 m. Rule 77 decreases the height of the wall between the patio and the street without decreasing the height of the wall between the patio and the neighboring lot. Rule 78 decreases the height of these walls evenly when the patio borders streets on both sides.

**Step 1.6: introduce openings.** The rules at this step pierce openings on the walls and introduce mullions in the openings. There are rules for piercing exterior openings (rules 79 – 93) and rules for piercing interior ones (rules 94 – 99). Rules 79 and 80 are the most important for placing exterior openings as they encode the basic principles used by Siza for the design of the front elevation. In frontyard houses, the strategy is as follows: each floor has two windows placed symmetrically in relation to each other, and the windows on the second floor are aligned with the windows on the first floor. Such a strategy holds even when the front walls of the first and second floors are on different planes. Rules 79 and 80 thus place two labelled axes \( e_1 \) on the first floor and
two labelled axes e2 on to the second floor, which are the labels that permit the application of rules 81 and 82. Rules 83–85 are bookkeeping rules as they erase openings on the front elevation that are hidden by the patio wall. Rules 86–93 specify how openings can be pierced in the front elevation in other situations or on other elevations. The design of the front elevation in backyard houses does not follow the same principles used in frontyard houses as the lateral corridor accessing the yard makes it difficult to design a symmetrical façade. The principle in this case is to accept asymmetry and use the remaining window-placement rules. Rules 94–97 pierce interior openings in orthogonal walls, whereas rules 98–99 (not shown) do so in diagonal walls. The derivation of the elevation proceeds with the design of the mullions using rules 100–107 (not shown). Interestingly, it follows the same compositional principle of rectangular dissections used in the design of the layout.

**Step 1.7: terminate.** The last step in the derivation of the first floor includes rules that delete unnecessary labels (rule 108) and change the state label from $F_1$ to $F_2$ (rule 109). The derivation then proceeds to the second floor.

**Stage 2: define the second floor**
To a certain extent, the derivation of the second floor is similar to that of the first floor. The rules are also very similar and so they will not be described in detail and they are not illustrated in figure 11, but the differences in derivations are pointed out below. When the actual derivation of the second floor starts, it has already inherited a series of labels from the derivation of the first floor. Such labels carry information that will be used to make new dissections, extend the staircase and the chimneys, and to place the openings. The first step of the derivation introduces the slab (rule 110) and the enclosing walls (rule 2), and adjust the wall thickness (rules 111–113). The second step replicates the dissections of the first floor, using the existing labels as markers. If the first dissection of the first floor was made 6 m away from the front of the lot, the corresponding dissection of the second floor can be made right above (rule 114) or 1 m backwards (rule 115), so that a verandah is created. The next dissections replicate the division of the first floor into functional zone dissections (rule 116). The subsequent dissections might (rule 117) or might not (rule 118) replicate the first floor, depending whether the number of required bedrooms ($n_B$) is equal to or larger than two. The third step of the derivation extends the staircase (rules 119 and 120) and defines the basic circulation scheme by creating a corridor perpendicular (rules 121 and 123) or parallel (rules 122 and 124) to the staircase. The choice between these two options depends on whether the lot borders a street or a house on the side where the corridor will be placed. The fourth step divides the remaining space into rooms or assigns a bedroom or a terrace to a room that resulted from the replication of the first floor (rules 125–137) There are some constraints to such operations: the rooms above the inside zone defined by the first dissection cannot become terraces and the remaining rooms can only become terraces if the layout has the required number of bedrooms. Other rules can be applied at this step to create corridors and bathrooms, or to extend an existing bedroom by dissecting another room or the end of a corridor. The last three steps of the derivation introduce the details (rules 138–146) and the openings (rules 81–97), and erase unnecessary labels (rule 147). The last rule (rule 148) changes the state label.

**Stage 3: define the terrace**
This stage has fewer steps than the previous two. The first step introduces the slab and encloses the terrace (rules 149 and 2). The second replicates the division of the lot into inside and outside zones (rule 150) and then into functional zones (rules 151–155).
Figure 12. Derivation of a t5 variation of a new backyard design. Because of space limitations, only the derivation of the plan and the main elevation of the first floor is shown. The rules applied during the derivation are shown below the arrows between design states. The complete generation (excluding three-dimensional viewpoints) may be found at http://www.civil.ist.utl.pt/~jduarte/epb/figure12.pdf.
1.4 Divide zones into rooms

<table>
<thead>
<tr>
<th></th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Locate bedroom/bathroom</td>
<td>Locate circulation</td>
<td>Locate kitchen</td>
<td>Locate laundry</td>
</tr>
</tbody>
</table>

1.5: Introduce details

<table>
<thead>
<tr>
<th></th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Connect living/circulation</td>
<td>Connect circulation</td>
<td>Connect living/circulation</td>
<td>Connect corridor/yard</td>
</tr>
</tbody>
</table>

Figure 12 (continued)
4.7 Derivation of a new design

Stiny and Mitchell (1978) listed three tests to confirm if a grammar has any explanatory or predictive value. First, it should reveal the common, underlying features of designs in the corpus—the descriptive test. Second, it should provide the criteria to determine whether a building is a design in the language—the analytic test. And, third, it should specify how to generate new designs in the language—the synthetic test. Several experiments were undertaken to test the grammar. The first was an attempt to generate all the designs in the corpus, starting with type $Ab$, the first mature design by Siza at Malagueira. Each additional design required fine-tuning of the grammar, resulting in the version shown in this paper. The second experiment was to derive a house by Siza, not in the original corpus. Siza designed this house in recent years, when some lots initially allocated for public facilities were converted to housing. With the exception of details to adjust the patio to a slightly different lot shape—trapezoid instead of rectangular—the grammar accounted for the derivation of the design. The third experiment was to generate a new backyard design, shown in figure 12. A backyard house was selected because Siza designed and built only one backyard housetype, and to design a new one represented a greater challenge. Figures 13 and 14 show the plans and the three-dimensional model of the new design, respectively. In the new design the staircase is placed in a different position (see figure 7). When the new design was shown to Siza amidst other Malagueira designs, he did not notice that it was not his own design. At some point, he seemed confused because he did not remember such a placement of the staircase. But then, he acknowledged its validity and agreed that the design was in the style. When he was told that it was not his design, he was surprised. Then, after a careful analysis of the design and an explanation of its derivation, he acknowledged that the grammar seemed to capture the style. In these three experiments the grammar
passed each of the three tests referred to above. Additional experiments were carried out with designs by other authors using the grammar, and Siza maintained his positive opinion. In some cases he mentioned that he would have not designed them for idiosyncratic reasons but he considered them to be in the style. In these experiments, authors were asked to design houses for given clients who later commented on whether the houses fulfilled their needs. These new experiments formed a new test, which I call the goal test. The ability of the grammar to generate customized designs was confirmed with this test. For more information on some of the experiments see Duarte (in press).

**Figure 13.** Plans, section, and elevations of four variations of a new backyard house.

**Figure 14.** Three-dimensional models of four variations of a new backyard house.
5 Discussion and conclusions

At the beginning of this paper three arguments were presented: (1) shape grammars can provide the technical apparatus to make Siza's design rules at Malagueira explicit; (2) shape grammars can be used to design customized Malagueira houses; and (3) a computer program encoding such a grammar would allow one to use Siza's design system more effectively.

The first argument is settled with the presentation of a grammar for the style of Malagueira courtyard houses. The use of simple compositional rules consisting of the dissection of rectangles determines the style. The grammar accounts for the generation of the thirty-five houses considered in the corpus, as well as new houses in the language. These new houses were generated both by the author of the grammar and by other authors following the grammar and Siza agreed that the houses were in the style. Therefore, the grammar successfully captures Siza's rules.

The second argument is also supported, to a certain extent, by the results of this work. The rules of the grammar include a set of constraints on design features that limit the ways in which rules can be applied so that a house matching given criteria is generated. The features are described in the label part of the rules and include functional and dimensional aspects, such as the number and type of spaces, adjacency relations, width, and area. A set of experiments undertaken to test the ability of the grammar to generate criteria-matching designs confirmed such an ability, but revealed the need to introduce some changes into the grammar to increase the opportunity for customizing the designs. These changes are expanding the criteria set a priori and opening the rules to include functional requirements not foreseen in the cultural context for which the houses were initially designed. For instance, in the grammar the laundry is next to the kitchen but one client wanted it next to the bathroom. Modifications to the shape grammar have been the goal of subsequent work.

The third argument will be fully demonstrated by the development of a computer program encoding the rules of the grammar. This program is now being developed, but it is possible to defend it on the basis of current experimental results and our knowledge of the computer. Designers who were asked to use the grammar to generate customized designs complained that it was difficult to remember client requirements while keeping track of all the technical details in the rules. However, once they had been instructed to use the grammar in the simplified way described in section 4.2, they were able to generate Malagueira designs but took some time to arrive at a solution that satisfied the client. The technical details make the grammar suitable for future computer encoding. By using the processing power of the computer it would be possible to accelerate the generation of designs.

Once I have succeeded in developing an interpreter for the Malagueira grammar capable of generating suitable designs, I will have shown that grammars and computers can be used to customize the design of Malagueira houses. Nevertheless, to show that they can be used to customize mass housing in general, I will have to show how one can design new housing grammars. I identify three different approaches. The first was proposed by Stiny (1980) for the development of grammars from scratch. In brief, this approach requires: the definition of a vocabulary of shapes; then, the development of a spatial relation by combining two of these shapes; next, the definition of additive and subtractive rules from these spatial relations; and finally the use of these rules to derive designs in the language. Stiny illustrated this approach with a grammar for Froebel's building blocks.

The second approach was developed by Knight (1986). It assumes the existence of a language of designs and proceeds by transforming this language into a new one.
Roughly, the steps involved are: get an existing grammar and the language it specifies; define the grammar in terms of spatial relations, and additive and subtractive rules; then, change some of the spatial relations, add or subtract rules, or alter the sequence of rule application; finally, use the transformed grammar to derive new designs. Knight showed the validity of this approach by deriving a grammar, for the language of Frank Lloyd Wright’s Usonian houses, from Koning and Eizenberg’s grammar for Wright’s Prairie houses.

The third approach is suggested by my experience in developing the Malagueira grammar, but also has its roots in the previous two approaches. It requires the development of a grammar from an existing set of designs and then enlarging the space of design solutions following the inner logic of the grammar. In the limit the existing set can be restricted to a single, prototypical design that one uses to sketch the grammar, which is then refined as more designs are generated. This was, to a certain extent, the process followed by Siza at Malagueira, who after having designed the first house (subtype Aa), designed the remainder as variations of the first. This was also the process followed in the development of the grammar as Siza’s prototypical design was used to sketch the grammar. Then, the grammar was refined to account for the generation of the remaining designs. This process showed that new designs are variations obtained by a different application of the rules in the sketched grammar, like the rule to allocate the staircase. Transformations of these rules or addition of new rules are, nevertheless, required to solve geometric problems posed by new rule applications (for example, a new orientation of the staircase) or to deal with new programmatic features (for example, the allocation of a laundry room). To a certain extent, this approach can be seen as a special case of the approach developed by Knight, with two differences. First, the initial design is not a historical precedent, but is designed as part of the methodology. The process through which this design is obtained can either be Stiny’s methodology, or the traditional process of designing. Second, one does not change the initial grammar to derive a new language of designs, but rather refines it to account for new designs in the same language. I expect to formalize this approach in future work.

In summary, this paper proposes the use of shape grammars as the formalism for developing an interactive computer system for designing customized mass housing based on the idea that shape grammars systematize the rules for generating designs within a language in such a way that it is possible to encode them into a computer program. Such an interactive system would enable greater user participation in the design process and, therefore, greater user satisfaction. Future work will focus on the refinement of the grammar, on the implementation of the corresponding computer program, and on the development of a methodology for creating new housing grammars. The bulk of work in these three areas will be concerned with the articulation of form and meaning to allow the generation of designs that match given criteria. Updated information on the Malagueira grammar, including models of houses in the corpus, and interactive three-dimensional derivations of existing and new houses, can be found on the web at http://www.civil.ist.utl.pt/~jduarte/malag/.

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