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Introduction

The Organizing Committee of SYMCOMP2013 – 1st International Conference on Symbolic and Algebraic Computation welcomes all the participants and acknowledge the contribution of the authors to the success of this event.

This First International Conference on Symbolic and Algebraic Computation, is promoted by APMTAC - Associação Portuguesa de Mecânica Teórica, Aplicada e Computacional and it was organized in the context of IDMEC/IST - Instituto de Engenharia Mecânica. With this ECCOMAS Thematic Conference it is intended to bring together academic and scientific communities that are involved with Algebraic and Symbolic Computation in the most various scientific areas

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To establish the state of the art and point out innovative applications and guidelines on the use of Algebraic and Symbolic Computation in the numerous fields of Knowledge, such as Engineering, Physics, Mathematics, Economy and Management,...

To promote the exchange of experiences and ideas and the dissemination of works developed within the wide scope of Algebraic and Symbolic Computation.

To encourage the participation of young researchers in scientific conferences.

To facilitate the meeting of APMTAC members (Portuguese Society for Theoretical, Applied and Computational Mechanics) and other scientific organizations members dedicated to computation, and to encourage new memberships.

We invite all participants to keep a proactive attitude and dialoguing, exchanging and promoting ideas, discussing research topics presented and looking for new ways and possible partnerships to work to develop in the future.

The Executive Committee of SYMCOMP2013 wishes to express his gratitude for the cooperation of all colleagues involved in various committees, from the Scientific Committee, Organizing Committee and the Secretariat. We hope everyone has enjoyed helping to birth this project, which we are sure will continue in the future. Our thanks to you all.

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AN EXPEDITIOUS METHODOLOGY TO CORRECT THE CLOSURE ERROR OF POINT CLOUDS REGISTRATION

Luís Mateus[1]*, Margarida Barbosa[2] and Victor Mota Ferreira[3]

1: ArcHC3D research group
CIAUD, Faculty of Architecture
Technical University of Lisbon
Rua Sá Nogueira, Pólo Universitário, Alto da Ajuda, 1349-055 Lx
lmmateus@fa.utl.pt, web: http://www.fa.utl.pt/~lmmateus

2: ArcHC3D research group
CIAUD, Faculty of Architecture
Technical University of Lisbon
Rua Sá Nogueira, Pólo Universitário, Alto da Ajuda, 1349-055 Lx
margarida.j.barbosa@fa.utl.pt

3: ArcHC3D research group
CIAUD, Faculty of Architecture
Technical University of Lisbon
Rua Sá Nogueira, Pólo Universitário, Alto da Ajuda, 1349-055 Lx
victor@fa.utl.pt, web: http://www.fa.utl.pt/~victor

Keywords: terrestrial laser scanning, closure error, correction, expeditious methodology, point clouds

Abstract *This paper describes an expeditious methodology to correct the closure error of the registration of a set of terrestrial laser scanning point clouds aligned sequentially. We start by defining what a closure error is and how it can be expressed in terms of matrix notation. Then we explain the valid setting from which a closure error can be obtained. After we describe the algorithm developed to dissipate the closure error. Finally we present an application of the algorithm and a comparison with an automatic procedure, describing the pros and cons of the adoption of the developed algorithm.*

1. INTRODUCTION

Laser Scanning Survey consists in the capture of multiple point clouds, by positioning a laser scanning system at various points of view, named stations, concerning the structure under study, recording as much information as possible.
Relative orientation of the point clouds, also called registration, is done through a common coordinate system, local or global, in order to produce a coherent three dimensional model. A local reference frame can be defined by the pose of a reference point cloud.
There are several ways to proceed to registration of the point clouds [1].
- Using a control survey, with total station, where at least 3 control points are common to each point cloud; the points can be materialized using targets and better results are obtained if automatic identification is performed.
- Using coded targets (spheres, chess boards, coloured circles, etc.) that are automatically identified by software and then used to calculate the required transformations needed to orient the point clouds, without any control survey.
- There are some references in literature about automatic registration of point clouds combining image processing techniques with the identification of geometric features in the point clouds [2]. However, this approach still didn't produce good results.
- Using natural points of the features recorded to proceed to an initial registration followed by an optimization procedure using an algorithm such as Iterative Closest Point, usually referred as ICP [3]. In this situation is necessary to be careful about the points that are picked by the operator.

1.1. Orienting a sequence of point clouds and the definition of closure error

To ensure accuracy in this process, particularly if the last of the aforementioned options is considered, it should be taken an approach from the general to the particular, orienting a sequence of point clouds that encloses the boundaries of the area to be recorded, and posteriorly filling out the spaces resulting from it.
To the line defined by the sequential positions of the origins of the local frames of each point cloud, is given the name of polygonal of point clouds. Usually, it is intended that this is a closed line (Fig 1).
When orienting a sequence of point clouds, without any control survey, there is always a drift that leads to the accumulation of errors in each orientation step. At the end, when the last point cloud (PC_n) of the polygonal is oriented, having the previous one (PC_{n_1}) as reference, there is a misfit between that point cloud (PC_n) and the first one (PC_1) of the set. This misfit can be measured if the first point cloud (PC_1) is oriented having the last one as reference(PC_n).
If the reference frame associated with the first point cloud is an identity matrix defining its pose, then the deviation is measured by the transformation matrix arising from the transformation of the last orientation procedure.

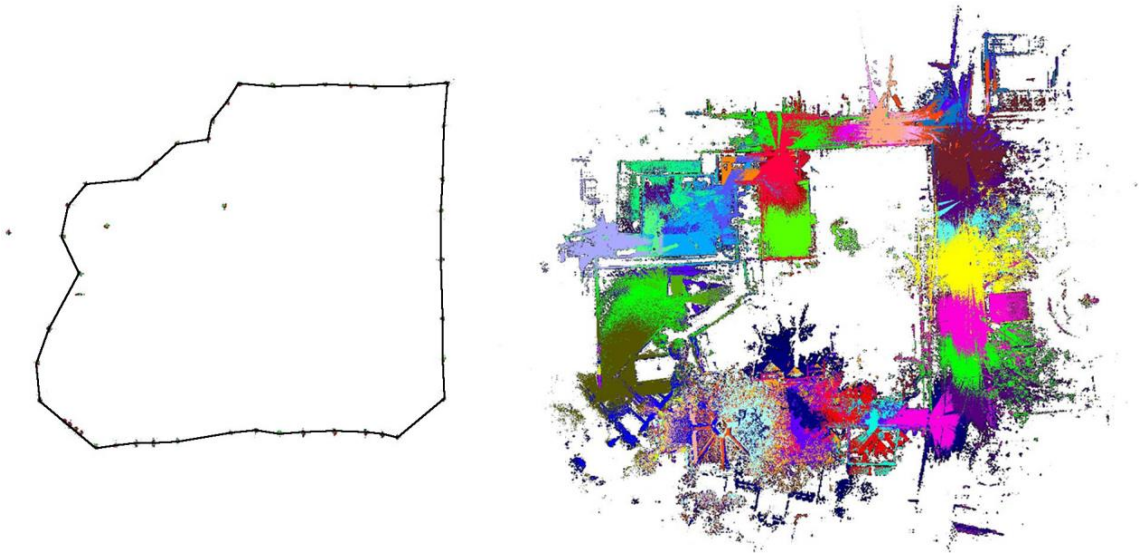

Figure 1: Closed polygonal of point clouds (left) and oriented point clouds (right).

The first point cloud pose is no longer defined by an identity matrix but rather by a matrix that defines what we call a closure error as it can be seen in expression (1).

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \Delta_x \\ a_{21} & a_{22} & a_{23} & \Delta_y \\ a_{31} & a_{32} & a_{33} & \Delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The sub matrix (2) represents the rotation component of the closure error.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

And the sub matrix (3) represents the translation component of the closure error.

$$\begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix} \quad (3)$$

The rotation component can also be described in axis-angle form.

$$\begin{matrix} A_X \\ A_Y \\ A_Z \\ \alpha \end{matrix} \tag{4}$$

Where A_X, A_Y and A_Z represent the components of the rotation axis and α represents the rotation angle in degrees.

Usually, this error can be dissipated by doing a bundle adjustment where there is many to many references between point clouds, as it can be done with Meshlab (http://meshlab.sourceforge.net/) or Polyworks (http://www.innovmetric.com/polyworks/3D-scanners/home.aspx). However, not all software tools allow this operation, as it is the case of JRC 3D Reconstructor (http://www.gexcel.it/en/software) or Faro Scene (http://www.faro.com/en-us/products/faro-software/scene/overview). It also has to be noticed that it implies bigger computational capacity.

The work presented in this paper describes a methodology to dissipate this error when these computational capacities or tools are not available. It further compares this approach with the bundle adjustment approach.

The method here described has some analogies with the "compass method" used it topography [4].

The dissipation of the closure error is required to produce a coherent 3D model without geometrical defects.

It is important to notice that if a closure error is to be used as a starting value to apply a correction method, no matter what, that should be a small value. One rule of thumb to decide if an error is acceptable is to consider the root mean square (rms) error of the registration process as reference. If a sequence of registrations is done, then the sum of the rms errors can be taken as reference. For example, if the rms error of a single registration is 2mm, then, after 10 consecutive registrations, the accumulated error should not exceed 20mm. And the rms error for a single registration should be smaller than the accuracy stated for the equipment in use for the average ranges in the scene recorded.

2. THE ALGORITHM TO DISSIPATE THE CLOSURE ERROR

The algorithm described in this paper is divided in two parts. First it is dissipated the translation component and them the rotation component. This dissipation process can also be called compensation [4].

2.1. Compensating the translation component of the closure error

Given a set of point clouds $(PC_1, PC_2, PC_3, \dots, PC_{n_1}, PC_n)$ oriented in sequence, each one having as reference the previous one, the pose of general point cloud (PC_i) can be described as square matrix.

$$\begin{bmatrix} i_{11} & i_{12} & i_{13} & \Delta_{x(i)} \\ i_{21} & i_{22} & i_{23} & \Delta_{y(i)} \\ i_{31} & i_{32} & i_{33} & \Delta_{z(i)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Similarly, the pose of the previous point cloud (PC_{i_1}) can be described the same way.

$$\begin{bmatrix} (i-1)_{11} & (i-1)_{12} & (i-1)_{13} & \Delta_{x(i-1)} \\ (i-1)_{21} & (i-1)_{22} & (i-1)_{23} & \Delta_{y(i-1)} \\ (i-1)_{31} & (i-1)_{32} & (i-1)_{33} & \Delta_{z(i-1)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The distance between the origins of the oriented point clouds (PC_i) and (PC_{i_1}) can be calculated by the expression (7).

$$D_{(i,i-1)} = \sqrt{(\Delta_{x(i)} - \Delta_{x(i-1)})^2 + (\Delta_{y(i)} - \Delta_{y(i-1)})^2 + (\Delta_{z(i)} - \Delta_{z(i-1)})^2} \tag{7}$$

Let the accumulated distances, resulting from the partial distances calculated with expression (7), be noted as follows.

$$\sum D_{ac} \tag{8}$$

Let the accumulated distances from the origin of the adopted reference frame to the origin of the oriented frame of the point cloud one wishes to compensate, (PC_i), resulting from the partial distances calculated with expression (7), be noted as follows.

$$\sum D_{corr}(i) \tag{9}$$

Then the compensation vector to be applied to point cloud (PC_i) is calculated proportionally to the ratio between these accumulated distances according to the following expression.

$$\begin{cases} C_{x(i)} = -\Delta_x \dfrac{\sum D_{Corr}(i)}{\sum D_{ac}} \\ C_{y(i)} = -\Delta_y \dfrac{\sum D_{Corr}(i)}{\sum D_{ac}} \\ C_{z(i)} = -\Delta_z \dfrac{\sum D_{Corr}(i)}{\sum D_{ac}} \end{cases} \tag{10}$$

After the compensation of the translation, the pose of point cloud (PC_i) is given by the following matrix.

$$\begin{bmatrix} i_{11} & i_{12} & i_{13} & (\Delta_{x(i)} + C_{x(i)}) \\ i_{21} & i_{22} & i_{23} & (\Delta_{y(i)} + C_{y(i)}) \\ i_{31} & i_{32} & i_{33} & (\Delta_{z(i)} + C_{z(i)}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

After applying the compensation to all the point clouds, the pose of (PC_1) should be given by the following matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

2.2. Compensating the rotation component of the closure error

After the compensation of the translation applied to all point clouds it remains the rotation component to be compensated. It follows a similar rule. Given the rotation component of the closure error as described by expression (4), the compensation rotation to be applied to point cloud (PC_i) is calculated proportionally to the ratio between the accumulated distances (9) and (8) according to the following expression.

$$R_i = -\alpha \frac{\sum D_{Corr}(i)}{\sum D_{ac}} \tag{13}$$

This value, expressed in degrees together with the components of the rotation axis A_X, A_Y and A_Z given by expression (4) can be used to compute the rotation matrix corresponding to the compensation. Then that matrix is multiplied by the sub matrix of the matrix (5) that describes the rotation of the pose of the point cloud(PC_i).
After the compensation of the translation, the pose of point cloud (PC_i) is given by the Let The pose of be described by the following matrix.

$$\begin{bmatrix} i'_{11} & i'_{12} & i'_{13} & (\Delta_{x(i)} + C_{x(i)}) \\ i'_{21} & i'_{22} & i'_{23} & (\Delta_{y(i)} + C_{y(i)}) \\ i'_{31} & i'_{32} & i'_{33} & (\Delta_{z(i)} + C_{z(i)}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

After applying the compensation to all the point clouds, the pose of (PC_1) should be given by the identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

At the end, the initial pose of (PC_1) is recovered.

3. PRACTICAL APPLICATION

The practical application consisted in orienting a set of 13 point clouds corresponding to the Church of "Santo André" in Mafra (Fig. 2).
The data was collected with a Faro Focus 3D phase based scanner.
It was done in two steps, in consecutive semesters, in academic context, in Faculty of Architecture – Technical University of Lisbon. The first step corresponded to the optional course of Architectural Photogrammetry (2nd cycle) during the second semester of 2011/2012 and the second step corresponded to the optional course of 3D Digitization (3rd cycle) during the first semester of 2012/2013.
In this application it was only considered the translation component of the closure error.

Figure 2: Images of the final oriented model.

3.1. First step – Architectural photogrammetry course 2011/2012 (2nd semester)

In the first year the students were asked to orient the point clouds in a specific sequence. All students prepared the point clouds independently (spurious data removal, deleting mobile objects such as cars or trees). It was strictly asked to orient each point cloud having the previous one in the sequence as reference, in order that comparable results were obtained. It was also asked to visually verify the results of each orientation step in order to avoid gross errors that would bias the analysis. Finally, students were asked to find a closure error, but not to correct it.

After the exercise was completed, it were selected those that presented smaller closure errors.

A total of four exercises were selected. The corresponding closure errors are displayed in table 1.

Closure error (m)	Student 1	Student 2	Student 3	Student 4	Average
X direction	-0,000315	-0,007811	0,019348	-0,007485	-0,003043
Y direction	-0,000867	0,016007	0,011365	0,005253	0,016435
Z direction	0,006563	0,001524	-0,037646	0,011785	-0,008307
3D	0,006628	0,017876	0,043826	0,014917	0,018665

Table 1: Average of closure errors.

Additionally, the translation component of each pose of the selected exercises was also averaged. These average values were considered as reference values to evaluate the exercises of the following course.

3.2. Second step – 3D Digitization course 2012/2013 (1st semester)

In this year, the same set of point clouds was used to develop a similar exercise. The difference was that students were asked to correct the translation component of the closure error in two different ways. Firstly, applying the method described in this paper, herein noted as our method (OM), and secondly, applying an automatic bundle adjustment (BA) process using the software Meshlab. The results of a single exercise are displayed in table 2.

Point Cloud ID	Compensation vectors Our method (OM)	Diference between OM and BA with Meshlab
Point Cloud 2	-0,000023 -0,000876 -0,001176	-0,001222 0,001539 0,000074
Point Cloud 3	-0,000041 -0,001542 -0,002068	-0,000907 0,002470 0,000028
Point Cloud 4	-0,000058 -0,002196 -0,002946	-0,000299 0,001977 -0,000850
Point Cloud 5	-0,000069 -0,002584 -0,003467	-0,000393 0,001082 -0,002610
Point Cloud 6	-0,000082 -0,003084 -0,004138	0,000054 0,001361 0,000274
Point Cloud 7	-0,000099 -0,003713 -0,004981	0,002137 -0,000411 0,002423
Point Cloud 8	-0,000120 -0,004523 -0,006068	0,004946 -0,001654 0,010056
Point Cloud 9	-0,000139 -0,005209 -0,006988	0,003035 -0,001023 0,012456
Point Cloud 10	-0,000157 -0,005911 -0,007931	0,002468 -0,001655 0,013371
Point Cloud 11	-0,000171 -0,006418 -0,008611	0,002176 -0,003037 0,011059
Point Cloud 12	-0,000196 -0,007374 -0,009893	0,002051 0,000272 0,008985
Point Cloud 13	-0,000222 -0,008334 -0,011181	0,000856 -0,002595 0,003607
Point Cloud 1	-0,000241 -0,009066 -0,012164	0,000059 -0,000703 -0,001200

Table 2. Comparison of both compensation methods. All values are in meters. These values correspond to the exercise of the student José Lima Ferreira.

In second column of table 2 we display the translation vectors resulting from the application of OM. In third column we display the differences between the translation vectors of the poses of all point clouds after both compensation methods were applied.
It can be observed that the bigger differences occurred in the vertical direction in four point clouds (values highlighted with red and yellow). However the values are relatively small, if we consider architectural applications, where 1cm tolerance is perfectly acceptable.
Notice that the blue highlighted values are symmetrical of the closure error initially obtained. Also notice that this value is comparable to the average values presented in table 1.

4. CONCLUSIONS

The paper described an expeditious methodology to correct the closure error of point clouds registration that can be easily applied with an excel spreadsheet, and used with good results for architectural applications where low computational capacity is available. However, in the practical application only one of the components of the method was applied, the compensation of the translation vector of the closure error.
Our method works in a linear way. In that sense the results are completely predictable and repeatable, what doesn´t happen with the automatic approach.

5. ENDNOTES

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